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SLOPE STABILITY MONITORING USING PRECISION AND RELIABILITY MEASURES DERIVED FROM LEAST-SQUARES

A.B. Kempe

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by

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ABSTRACT

In this paper, the method of least squares is described as it applies to the reduction of redundant observations in a survey network. The method is discussed in relation to its application to slope stability monitoring in surface mines.

Precision estimates for the unknown parameters are derived, reflecting the amount of trust that can be placed on the values determined for these parameters. Presented are reliability measures, which give insight into the controllability of observations in the network and the sensitivity of the network to blunders in the observations. In sum, precision and reliability measures are seen as a means of locating weak areas in the network, enabling blunder detection, and providing useful knowledge to the surveyor to protect oneself against the presence of observational errors.

Numerical results from a least squares adjustment program are also illustrated and discussed in an attempt to further clarify the theoretical ideas presented.

SURVEILLANCE DE LA STABILITÉ DES PENTES PAR DES MESURES DE PRÉCISION ET DE FIABILITÉ ÉTABLIES AU MOYEN DE LA MÉTHODE DES MOINDRES CARRÉS

par

A.B. Kempe

RÉSUMÉ

Le document porte sur la méthode des moindres carrés telle qu'appliquée à la réduction des observations superflues d'un réseau de surveillance. La méthode est étudiée en fonction de son application à la surveillance de la stabilité des pentes des mines à ciel ouvert.

Les auteurs utilisent la méthode pour faire une évaluation précise des paramètres inconnus qui peuvent influer sur la fiabilité des valeurs établies pour ces mêmes paramètres. Les mesures de fiabilité qui sont présentées dans ce rapport donnent un aperçu de la mesure dans laquelle il est possible de contrôler les observations du réseau et de la susceptibilité de celui-ci aux erreurs. En définitive, les mesures de précision et de fiabilité sont perçues comme ur moyen pour localiser les points faibles du réseau et détecter les erreurs. Le surveillant peut, de cette façon, obtenir des renseignements utiles lui permettant de prévenir les erreurs d'observation.

Les résultats numériques du programme des moindres carrés sont également présentés et étudiés en vue de clarifier davantage les idées théoriques présentées dans ce document.

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INTRODUCTION

The detection and measurement of slope displacement in open pit mines remains primarily the domain of terrestrial survey methods and equipment. Geotechnical instruments such as strain or borehole extensometers and inclinometers often have a high resolution but yield only relative, local, information. To have an absolute reference frame, enabling the determination of extent, rate, and direction of movement, a survey network is necessary.

The procedure is to determine movements by comparing the coordinates determined for each epoch of the survey network. The significance of the movements depends on the precision of the observations and their assumed normal distribution. Thus any outliers in the observations, or any systematic effects on the observations, could be incorrectly interpreted as significant movements. Results from many slope monitoring campaigns , illustrating immense fluctuations in movements of points in both magnitude and direction between recent epochs, confirm these incorrect interpretations (see Table 1, Appendix). Also, these inconsistent results illustrate the unreliability of present observation procedures and the need for enhanced mathematical processing of the observations.

In this paper, the method of least squares is presented as it applies to survey networks. The method is shown as a means of solving an over-determined equation system, which minimizes corrections to the observables. Further, the method is shown to yield precision estimates for the unknowns, and reliability measures which give insight into the controllability and sensitivity of the survey network.

THE METHOD OF LEAST-SQUARES

The relationship between the unknown parameters (eg. coordinates) and the survey observations made in the field, is through the formulation of the mathematical model. With the specification of the mathematical model, estimates of values for the unknowns and their

attained accuracy can be obtained through application of a least squares adjustment. An adjustment is required due to the fact that the math model is usually over-determined. That is, more observations are taken than are necessary to uniquely determine values for the unknovns. However due to the stochastic nature of the observations, the observations will not uniquely define the unknovns because of a lack of perfect fit to the math model.

A least squares adjustment provides the means to solve a redundant, or over-determined equation system. The least squares process is based on the condition that the weighted sum of the squares of the estimated residuals (corrections to the observables) is minimized. This condition is given as:

$$\hat{r}^T C_l^{-1} \hat{r} \quad \longrightarrow \quad \text{minimum} \qquad (1)$$

where:

- $\hat{r}$ is the vector of estimated residuals, or corrections to the observations l, and
- C_l is the covariance matrix of the observations describing the accuracy of the observation vector l.

The general form of the mathematical model is given in parametric form as:

$$l = f(x) \qquad (2)$$

where x represents the estimates for the unknovn parameters. The mathematical model for equation (2) is, in general, nonlinear. The least squares method, however, solves systems of linear equations and thus the math model must be linearized. This is done using a linear Taylor's series and neglecting higher order terms as follovs:

$$l = f(x) \approx f(x^\circ) + \left.\frac{df}{dx}\right|_{x=x^\circ} (x-x^\circ) \qquad (3)$$

Thus the linear parametric model is:

$$l = Ax + V \qquad (4)$$

with

$$A = \left.\frac{df}{dx}\right|_{x=x^\circ} \quad \text{the design matrix} \qquad (5)$$

and

$$W = f(x^\circ) - l^\circ \qquad (6)$$

where

x° is the approximate values of the unknowns x;
l° is the observed values of the observables l.

Noting that higher order terms of the Taylor series are neglected, the solution must be iterated to get a better approximation for the unknown parameters.

The differential form of equation (4) is given as:

$$\hat{r} = A\hat{\delta} + W \ , \qquad (7)$$

where $\hat{\delta}$ is the least squares estimate for the corrections to x°. The solution for the corrections to the unknowns is given by the following equation:

$$\hat{\delta} = -(A^T C_l^{-1} A)^{-1} A^T C_l^{-1} W \ , \qquad (8)$$

and the least squares estimate (adjusted value) of the unknowns is given by:

$$\hat{x} = x^\circ + \hat{\delta} \ . \qquad (9)$$

Iteration involves solving equation (8), correcting the approximate values for x° with $\hat{\delta}$, then reforming the design matrix A of equation (7), and repeating the computation of $\hat{\delta}$. This procedure should be repeated until the corrections to the unknown parameters $\hat{x}$ become negligible.

From the "final" estimated residual vector $\hat{r}$ (equation (7)), the adjusted value of the observables $\hat{l}$ is computed as:

$$\hat{l} = l^\circ + \hat{r} \ . \qquad (10)$$

PARAMETER PRECISION

To adequately assess the quality of the determination of the parameters (eg. coordinates), statements pertaining to the precision of the determination are required. The covariance matrix resulting from a least squares adjustment contains a complete statistical statement about the unknown parameters. The covariance matrix for the unknown parameters $C_{\hat{x}}$ is given as:

$$C_{\hat{x}} = (A^T C_l^{-1} A)^{-1} \qquad (11)$$

From the $C_{\hat{x}}$ matrix, confidence regions may be determined which reflect the amount of trust that can be placed on the estimated values of the unknown parameters. For 3-dimensional adjustments, the confidence region is represented by an ellipsoid. Point confidence ellipsoids derived from $C_{\hat{x}}$ define the precision of the determination of a network station relative to the definition of the origin of the network. Thus, the precision of the unknown points are characterized by the size and orientation of the confidence ellipsoid relative to the fixed points in the network.

RELIABILITY (INTERNAL)

The quality of the determination of the unknown points in a network is a function of both precision and reliability. Through random error propagation, the representation of the precision of the results may be obtained, for given observable precision and network geometry, in the form of confidence regions (discussed above). The reliability of the network, describes the effect of true error propagation, both random and non-stochastic, on the accuracy determination of the network. Internal reliability measures give an estimate of the size an observational error must be before it will be detected through examination of the residuals from an adjustment.

Reliability measures give an insight to the weak areas of the network. Thus, if the weak areas of the network cannot be improved, due to set-up location limitations etc, then the surveyor has the knowledge that extra care or procedures must be used in these areas to protect oneself against the presence of observational errors.

In the presence of non-stochastic errors Δl in the observation l°, a shifted vector l' results as (Teskey et al., 1982):

$$l' = l^{\circ} + \Delta l \qquad (12)$$

The introduction of a shifted observation vector l' into a least squares adjustment produces shifts in the estimated values of the correction vector $\hat{\delta}$ and the residuals $\hat{r}$. For the parametric case the shifted estimates δ' and r'are given as:

$$\delta' = \hat{\delta} + \Delta\delta \qquad (13)$$

and

$$r' = A\delta' + W' \qquad (14)$$

with

$$\hat{\delta} = -(A^T C_l^{-1} A)^{-1} A^T C_l^{-1} W \qquad (15)$$

$$\Delta\delta = (A^T C_l^{-1} A)^{-1} A^T C_l^{-1} \Delta l \qquad (16)$$

$$W' = W - \Delta l. \qquad (17)$$

Thus, the shifted residual vector r' is computed as (Mackenzie, 1985):

$$r' = -A(A^T C_l^{-1} A)^{-1} A^T C_l^{-1} W' + W' \qquad (18)$$

which may be revritten as:

$$r' = C_{\hat{r}} C_l^{-1} W - C_{\hat{r}} C_l^{-1} \Delta l, \qquad (19)$$

where

$$C_{\hat{r}} = C_l - C_{\hat{l}} \qquad (20)$$

$$C_{\hat{l}} = A C_{\hat{x}} A^T. \qquad (21)$$

The matrix product $C\hat{r}C_l^{-1}$ of equation (19) is symmetric and idempotent (Mikhail, 1976). Due to this idempotent property, the trace (sum of the diagonal elements) of $C\hat{r}C_l^{-1}$ is equal to its rank, the redundancy of the network. Each diagonal element of $C\hat{r}C_l^{-1}$, denoted here as r_i , then represents the contribution of the ith observation to network redundancy (Mackenzie, 1985). The size of this contribution is a factor of network geometry and observational accuracy. Individual r_i 's, or **redundancy numbers**, give an insight into the controllability of observations. An observation is said to be fully controlled if one hundred percent of any observational error appears in the estimated residual. This would occur only in observations between fixed stations, in the absence of nuisance parameters, reflecting the fact that all of the observational error would appear in the estimated residual.

Residuals are not true errors, but rather estimates of observational errors subject to geometrical and computational weaknesses in the network adjustment process. Redundancy numbers reflect the amount of observational error recovered in the least squares process, as represented by the estimated residual.

Another useful quantity in analyzing survey networks is the **minimally detectable blunder** $\nabla_o l_i$. These values represent the size a blunder in an observation must be before leading to the rejection of the test on its standardized residual. They give insight into the sensitivity of the network to the presence of non-stochastic errors. The derivation of the expression for $\nabla_o l_i$ is quite involved and thus only the final expression is presented here. Interested readers are referred to (Mackenzie, 1985) for the derivation.

The expression for $\nabla_o l_i$ is given (for uncorrelated observations) as:

$$\nabla_o l_i = \sigma_{l_i} \sqrt{\lambda_o / r_i} \qquad (22)$$

where,

σ_{l_i} is the standard deviation of the ith observation,
r_i is the redundancy number of the ith observation,

λ_o is the bound for the separation of the distribution of the tested quantities in the presence of non-stochastic errors.

DISCUSSION OF RESULTS (SIMULATED)

Utilizing a local 3-dimensional least squares adjustment program, which incorporates the equations previously described, data from a 3-D network survey was processed for illustration purposes. Output from the program is shown in the appendix, and the following is a discussion of the results to further illustrate the ideas presented above.

Table 2 contains the initial approximate coordinates of the unknowns as input into the program. Table 3 contains the final adjusted coordinates as output from the program. Note that differences as large as 3 meters exist between the northings and eastings of stations after the adjustment, while differences up to 5 meters exist in elevation. This indicates that the accuracy of initial approximations to the unknowns is not critical, however "reasonable" estimates are recommended to reduce the number of program iterations and ensure successful convergence of the solution.

Table 5 contains the 99% point confidence analysis. Shown are the 3-D, 2-D, and 1-D confidence regions, reflecting the amount of trust that can be placed on the estimated values of the unknown parameters. 3-dimensional confidence ellipsoids are given in terms of the lengths of the three axes and their orientation to the three planes of reference. The 3-D ellipsoids are further reduced to a 2-D projected confidence ellipse and a vertical confidence bar. These values are quite self-explanatory.

Table 6 contains the internal reliability analysis. Shown is the residual, redundancy number, and minimally detectable blunder for each observation in the network, which give insight to the weak areas of the network. Examination of the results reveals that observations with lov redundancy numbers (weak geometric connection to the network) have

considerably larger minimally detectable blunders, while observations with high redundancy numbers (strong geometric connection to the network) are much more sensitive to observational blunders

Table 7 contains the internal reliability analysis for the same network but an observational error of 10" of arc was placed on the direction observation from station "F2" to station "C". This observation is characterized by a relatively high redundancy number (73%) and high sensitivity to blunders (8.1" of arc). Even with such a small error introduced, a considerable increase in the size of the residual is evident (from 0.74" of arc to 8.02" of arc). This residual is nov the largest of those for all directions and could easily be spotted as a blunder simply by examination of the residuals. Note also that the adjusted coordinates for station "C" derived using the data set containing the blunder (Table 4), differ by as much as 8mm from those using the data set without the blunder (Table 3). Had this been a subsequent epoch in a slope monitoring survey, the surveyor without these tools would not have detected the blunder and this difference could have been incorrectly interpreted as significant movement. Internal reliability measures provide the means for surveyors to detect such blunders and either eliminate them (by re-observing, or giving the observation negligible weight in the adjustment), or simply recognize their existence and interpret the results accordingly.

CONCLUSIONS AND RECOMMENDATIONS

In this paper, tools relevant to 3-D survey network adjustment and analysis have been derived and described. The objective of this paper, namely to describe the method of least-squares as it applies to survey networks and illustrate its usefulness in precision and reliability determination, has been satisfied to a level of detail suitable for anyone with knowledge in surveying and its practices. The aim is not to understand the matrix algebra and derivations presented, but rather to understand the resulting quantities and how they can assist in analyzing survey networks.

The method of least-squares must certainly be incorporated into the processing of observations from slope monitoring surveys. The method gives precision estimates for the unknowns, presently non-existent in most monitoring surveys, and reliability measures to enable blunder detection and the location of weak areas of the network.

Further research into observation procedures, namely the number of observations necessary to obtain a desired precision and the actual observation sequence, must certainly be undertaken. Field observations to existing monitoring targets in the mines should be made and subsequent processing of the observations using a 3-D adjustment program completed. Through this "hands-on" research, observation procedures can be optimized, program inefficiencies eliminated, and further need for enhancement recognized.

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TABLE 1: Slope Monitoring Report for Individual Target Point
(Movements shown are from successive epochs)

DATE: dd/mm/yy	TIME:	NORTHING:	EASTING:	ELEVATION:	MOVE: (mm.)	VELO: (/day)	AZM: (degrees)	VERT:
18/09/86	14:00	46619.563	49185.743	1753.375	0.	0.	0.	0.
20/09/86	13:30	46619.563	49185.746	1753.376	3.	2.	90.	18.
21/09/86	14:15	46619.557	49185.750	1753.375	7.	7.	146.	-8.
22/09/86	08:20	46619.563	49185.748	1753.381	9.	12.	342.	43.
23/09/86	12:50	46619.563	49185.744	1753.371	11.	9.	270.	-68.
24/09/86	11:30	46619.571	49185.744	1753.372	8.	9.	0.	7.
03/10/86	13:45	46619.609	49185.750	1753.359	41.	4.	9.	-19.
05/10/86	10:40	46619.619	49185.753	1753.360	10.	6.	17.	5.
09/10/86	11:00	46619.633	49185.743	1753.358	17.	4.	324.	-7.
11/10/86	10:20	46619.637	49185.746	1753.360	5.	3.	37.	22.
15/10/86	13:00	46619.638	49185.747	1753.354	6.	1.	45.	-77.
18/10/86	13:20	46619.641	49185.749	1753.354	4.	1.	34.	0.
23/10/86	12:00	46619.661	49185.755	1753.345	23.	5.	17.	-23.
24/10/86	14:00	46619.658	49185.758	1753.348	5.	5.	135.	35.
26/10/86	11:00	46619.661	49185.754	1753.350	5.	3.	307.	22.
29/10/86	11:30	46619.663	49185.755	1753.344	6.	2.	27.	-70.
05/11/86	15:20	46619.679	49185.755	1753.340	16.	2.	0.	-14.
09/11/86	08:40	46619.723	49185.763	1753.319	49.	13.	10.	-25.
10/11/86	09:15	46619.723	49185.768	1753.322	6.	6.	90.	31.
15/11/86	14:30	46619.821	49185.780	1753.264	115.	22.	7.	-30.
17/11/86	10:10	46619.826	49185.780	1753.265	5.	3.	0.	11.
21/11/86	15:00	46619.842	49185.787	1753.260	18.	4.	24.	-16.
22/11/86	11:00	46619.853	49185.790	1753.255	12.	15.	15.	-24.
22/11/86	14:25	46619.866	49185.793	1753.246	16.	113.	13.	-34.
24/11/86	14:30	46619.877	49185.798	1753.241	13.	7.	24.	-22.

TABLE 2 : INITIAL APPROXIMATE COORDINATES

STATUS	STA	NORTHING (m)	EASTING (m)	ELEVATION (m)
FX	F1	3812.107	1040.273	1022.141
FX	F2	2423.597	2214.550	1074.465
FX	F3	3820.126	3411.293	1089.948
FX	F4	2026.560	3198.316	2096.808
UK	A	3478.916	1499.993	1201.933
UK	B	3496.178	2313.772	1222.366
UK	C	3420.391	3006.054	1210.597
UK	D	3207.799	845.036	1262.884
UK	E	2819.724	1576.065	1289.061
UK	F	2814.059	2392.213	1246.726
UK	G	2818.718	3033.316	1272.810
UK	H	2116.390	1051.481	1063.731
UK	I	1535.474	2105.791	1092.539

TABLE 3 : ADJUSTED COORDINATES

STATUS	STA	NORTHING (m)	EASTING (m)	ELEVATION (m)
UK	A	3479.621	1501.710	1203.045
UK	B	3493.911	2316.396	1222.556
UK	C	3417.441	3003.513	1213.869
UK	D	3210.737	845.760	1261.895
UK	E	2822.924	1579.950	1289.397
UK	F	2814.177	2389.634	1249.563
UK	G	2817.435	3032.329	1275.528
UK	H	2118.678	1054.133	1064.864
UK	I	1536.533	2104.076	1087.918

TABLE 4 : ADJUSTED COORDINATES (Derived using data set containing 10" blunder in direction to station "C")

STATUS	STA	NORTHING (m)	EASTING (m)	ELEVATION (m)
UK	A	3479.621	1501.711	1203.044
UK	B	3493.911	2316.396	1222.556
UK	C	3417.449	3003.505	1213.868
UK	D	3210.737	845.761	1261.896
UK	E	2822.925	1579.952	1289.398
UK	F	2814.176	2389.635	1249.563
UK	G	2817.435	3032.330	1275.527
UK	H	2118.678	1054.133	1064.864
UK	I	1536.541	2104.073	1087.918

TABLE 5 : 99 % POINT CONFIDENCE ANALYSIS

APRIORI VARIANCE FACTOR = 1.000 EXPANSION FACTORS = [2.576 (1D), 3.035 (2D), 3.368 (3D)].

STATION NAME	3 DIMENSIONAL ELLIPSOIDS AXES METERS	ORIENTATION TO RESPECTIVE PLANES DEGREES NE	EH	NH	2-D PROJECTED CONFIDENCE ELLIPSES A METERS	B	AZIMUTH OF A DEGREES	CONF INT ELEV METERS
A	A= 0.019	37	31	38	0.016	0.005	44	0.012
	B= 0.013	50	37	14				
	C= 0.004	14	38	49				
B	A= 0.026	77	9	9	0.010	0.006	128	0.020
	B= 0.009	13	33	54				
	C= 0.006	2	55	35				
C	A= 0.019	17	39	46	0.017	0.004	133	0.012
	B= 0.016	70	19	6				
	C= 0.004	9	45	43				
D	A= 0.025	4	20	70	0.022	0.008	108	0.016
	B= 0.023	69	18	11				
	C= 0.005	21	63	17				
E	A= 0.022	43	29	33	0.016	0.004	45	0.013
	B= 0.012	46	36	22				
	C= 0.005	8	40	49				
F	A= 0.016	35	32	38	0.013	0.005	122	0.010
	B= 0.012	48	1	42				
	C= 0.003	20	58	24				
G	A= 0.024	62	18	21	0.013	0.005	133	0.017
	B= 0.011	28	42	35				
	C= 0.005	4	43	47				
H	A= 0.060	0	1	89	0.054	0.010	89	0.006
	B= 0.011	2	88	1				
	C= 0.008	88	2	0				
I	A= 0.142	0	83	7	0.128	0.026	7	0.006
	B= 0.029	0	7	83				
	C= 0.007	90	0	0				

TABLE 6 : INTERNAL RELIABILITY ANALYSIS

STATUS	FROM	TO	RESIDUAL (sec/km)	RED No	MIN DET BLUNDER (sec/km)
DR	F1	D	0.314	1	90.1887
DR	F1	H	0.203	9	23.3035
DR	F1	I	0.723	23	14.4448
DR	F1	E	-1.215	62	8.8806
DR	F1	A	-1.085	33	12.8121
DR	F1	B	0.678	70	8.2861
DR	F1	F2	0.249	71	8.1604
DR	F2	F1	0.398	88	7.3280
DR	F2	D	3.403	85	7.4922
DR	F2	A	-1.865	76	7.9888
DR	F2	E	0.809	51	10.0132
DR	F2	I	-0.558	14	19.1758
DR	F2	F	-0.882	23	16.1289
DR	F2	B	-2.650	83	7.6888
DR	F2	F4	2.878	88	7.3668
DR	F2	G	-1.776	72	8.2868
DR	F2	C	0.740	73	8.1448
DR	F2	F3	-0.797	88	7.3275
DR	F3	F2	0.051	88	7.3161
DR	F3	A	-1.767	86	7.4348
DR	F3	B	0.425	83	7.5479
DR	F3	E	4.430	87	7.3679
DR	F3	H	-0.594	76	7.8549
DR	F3	F	-2.904	81	7.6752
DR	F3	C	-1.344	20	16.6746
DR	F3	I	0.969	47	10.0029
DR	F3	G	1.421	77	7.9711
DR	F3	F4	-0.850	88	7.3169
SD	F1	D	0.182	1	62.7073
SD	F1	H	-0.091	1	135.4576
SD	F1	E	1.623	19	18.2670
SD	F1	A	0.054	7	15.3144
SD	F2	A	0.301	40	13.8824
SD	F2	E	0.461	7	20.5234
SD	F2	F	0.074	4	16.1977
SD	F2	B	-0.169	6	31.3618
SD	F2	G	0.646	7	24.7364
SD	F2	C	7.125	58	11.5003
SD	F3	B	-0.793	5	35.3198
SD	F3	F	0.230	50	13.8990
SD	F3	G	0.972	10	23.3508
SD	F3	C	1.492	11	11.9210
ZD	F1	D	0.545	2	50.4234
ZD	F1	H	-0.797	98	6.9744
ZD	F1	I	-1.228	99	6.8968
ZD	F1	E	1.371	80	7.8031
ZD	F1	A	0.004	41	11.4173
ZD	F2	A	0.408	87	7.4380
ZD	F2	E	1.101	55	9.6215
ZD	F2	F	-0.307	35	12.7582
ZD	F2	B	-1.629	49	10.0098
ZD	F2	G	1.053	51	9.8917
ZD	F2	C	4.250	87	7.4521
ZD	F3	B	1.197	54	9.4557
ZD	F3	F	1.785	93	7.1605
ZD	F3	G	0.483	64	8.7239
ZD	F3	I	0.647	99	6.8867
ZD	F3	C	2.453	40	11.5573
DH	F1	H	4.887	52	16.7326
DH	H	I	2.146	22	16.7173
DH	I	F2	2.967	30	16.4576

TABLE 7 : INTERNAL RELIABILITY ANALYSIS (derived from data set containing 10" blunder in direction F2 -> C)

STATUS	FROM	TO	RESIDUAL (sec/mm)	RED No	MIN DET BLUNDER (sec/mm)
DR	F1	D	0.228	1	90.1882
DR	F1	H	0.332	9	23.3035
DR	F1	I	0.815	23	14.4449
DR	F1	E	-1.386	62	8.8806
DR	F1	A	-1.382	33	12.8121
DR	F1	B	0.815	70	8.2861
DR	F1	F2	0.393	71	8.1604
DR	F2	F1	-0.580	88	7.3280
DR	F2	D	2.472	85	7.4922
DR	F2	A	-2.649	76	7.9888
DR	F2	E	0.316	51	10.0132
DR	F2	I	-0.631	14	19.1759
DR	F2	F	-1.226	23	16.1290
DR	F2	B	-3.580	83	7.6888
DR	F2	F4	1.900	88	7.3668
DR	F2	G	-2.622	72	8.2868
DR	F2	C	8.015	73	8.1448
DR	F2	F3	-1.775	88	7.3275
DR	F3	F2	-0.327	88	7.3161
DR	F3	A	-2.069	86	7.4348
DR	F3	B	0.031	83	7.5479
DR	F3	E	4.106	87	7.3679
DR	F3	H	-0.973	76	7.8549
DR	F3	F	-3.466	81	7.6752
DR	F3	C	2.154	20	16.6740
DR	F3	I	1.100	47	10.0029
DR	F3	G	0.940	77	7.9711
DR	F3	F4	-1.228	88	7.3169
SD	F1	D	0.132	1	62.7073
SD	F1	H	-0.154	1	135.4576
SD	F1	E	1.314	19	18.2669
SD	F1	A	0.102	7	15.3144
SD	F2	A	0.514	40	13.8824
SD	F2	E	0.371	7	20.5233
SD	F2	F	0.052	4	16.1977
SD	F2	B	-0.137	6	31.3618
SD	F2	G	0.696	7	24.7364
SD	F2	C	7.836	58	11.5004
SD	F3	B	-1.003	5	35.3198
SD	F3	F	-0.084	50	13.8990
SD	F3	G	1.171	10	23.3508
SD	F3	C	1.683	11	11.9211
ED	F1	D	0.396	2	50.4233
ED	F1	H	-0.797	98	6.9744
ED	F1	I	-1.246	99	6.8968
ED	F1	E	1.185	80	7.8031
ED	F1	A	0.153	41	11.4173
ED	F2	A	0.476	87	7.4380
ED	F2	E	0.834	55	9.6214
ED	F2	F	-0.449	35	12.7581
ED	F2	B	-1.669	49	10.0098
ED	F2	G	1.166	51	9.8917
ED	F2	C	4.456	87	7.4521
ED	F3	B	1.154	54	9.4557
ED	F3	F	1.739	93	7.1605
ED	F3	G	0.583	64	8.7239
ED	F3	I	0.647	99	6.8867
ED	F3	C	2.897	40	11.5574
DH	F1	H	4.887	52	16.7326
DH	H	I	2.145	22	16.7173
DH	I	F2	2.968	30	16.4576